

NONLINEAR DYNAMICS AND SYSTEMS THEORY

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CONTENTS

PERSONAGE IN SCIENCE

Professor V.M. Starzhinskii. To the 100th Birthday Anniversary.....1
A.A. Martynyuk, N.A. Izobov, A.G. Mazko and V.I. Zhukovskii

Generalized Monotone Method for Nonlinear Caputo Fractional
 Impulsive Differential Equations.....3
Y.Bai and A.S. Vatsala

Control Design for Non-Linear Uncertain Systems via Coefficient
 Diagram Method: Application to Solar Thermal Cylindrical
 Parabolic Trough Concentrators 21
*Z. Fenhouche, M. Chakir, O. Benzineb, M.S. Boucherit
 and M. Tadjine*

Adaptive Sliding Mode Control Synchronization of a Novel, Highly
 Chaotic 3-D System with Two Exponential Nonlinearities..... 38
F. Hannachi

Motion Control Design of UNUSAITs AUV Using Sliding PID..... 51
T. Herlambang, S. Subchan, H. Nurhadi and D. Adzkiya

Alternative Legendre Functions for Solving Nonlinear Fractional
 Fredholm Integro-Differential Equation 61
Khawlah H. Hussain

On the Boundedness of a Novel Four-Dimensional Hyperchaotic System..... 72
S. Rezzag

Periodic Solutions in Non-Homogeneous Hill Equation 78
A. Rodriguez and J. Collado

Estimates of Accuracy for Asymptotic Soliton-Like Solutions to the
 Singularly Perturbed Benjamin-Bona-Mahony Equation..... 92
V. H. Samoilenko, Yu. I. Samoilenko and L. V. Vovk

Oscillation Criteria for Delay Equations with Several Non-Monotone
 Arguments..... 107
*I. P. Stavroulakis, Zh. Kh. Zhumussova, L. Kh. Zhumussova
 and K. Dosmagulova*

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Singularly Perturbed Benjamin-Bona-Mahony Equation 92
V.H. Samoilenko, Yu. I. Samoilenko and L. V. Vovk
- Oscillation Criteria for Delay Equations with Several Non-Monotone
Arguments 107
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Oscillation Criteria for Delay Equations with Several Non-Monotone Arguments

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Abstract: Consider the first-order linear differential equation with several retarded arguments $x'(t) + \sum_{i=1}^m p_i(t)x(\tau_i(t)) = 0$, $t \geq t_0$, where the functions $p_i, \tau_i \in C([t_0, \infty), \mathbb{R}^+)$, for every $i = 1, 2, \dots, m$, $\tau_i(t) \leq t$ for $t \geq t_0$ and $\lim_{t \rightarrow \infty} \tau_i(t) = \infty$. In this paper we review the most interesting sufficient conditions under which all solutions oscillate. An example illustrating the results is given.

Keywords: *oscillation; retarded; differential equations; non-monotone arguments.*

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1 Introduction

Consider the first-order linear differential equation with several non-monotone retarded arguments

$$x'(t) + \sum_{i=1}^m p_i(t)x(\tau_i(t)) = 0, \quad t \geq t_0, \quad (1.1)$$

where the functions $p_i, \tau_i \in C([t_0, \infty), \mathbb{R}^+)$, for every $i = 1, 2, \dots, m$, (here $\mathbb{R}^+ = [0, \infty)$), $\tau_i(t) \leq t$ for $t \geq t_0$ and $\lim_{t \rightarrow \infty} \tau_i(t) = \infty$.

Let $T_0 \in [t_0, +\infty)$, $\tau(t) = \min \{\tau_i(t) : i = 1, \dots, m\}$ and $\tau_{-1}(t) = \sup \{s : \tau(s) \leq t\}$. By a solution of the equation (1.1) we understand a function $x \in C([T_0, +\infty), \mathbb{R})$, continuously differentiable on $[\tau_{-1}(T_0), +\infty)$ and that satisfies (1.1) for $t \geq \tau_{-1}(T_0)$. Such a solution is called *oscillatory* if it has arbitrarily large zeros, and otherwise it is called *non-oscillatory*.

For the general theory the reader is referred to [9, 11, 12, 17].

The oscillatory behavior of functional differential equations has been the subject of many investigations. See, for example, [1–20] and the references cited therein.

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